Demonstrating Integrity for Loran Cycle Selection Using Weighted Sum Squared Error (WSSE) Statistic

International Loran Association

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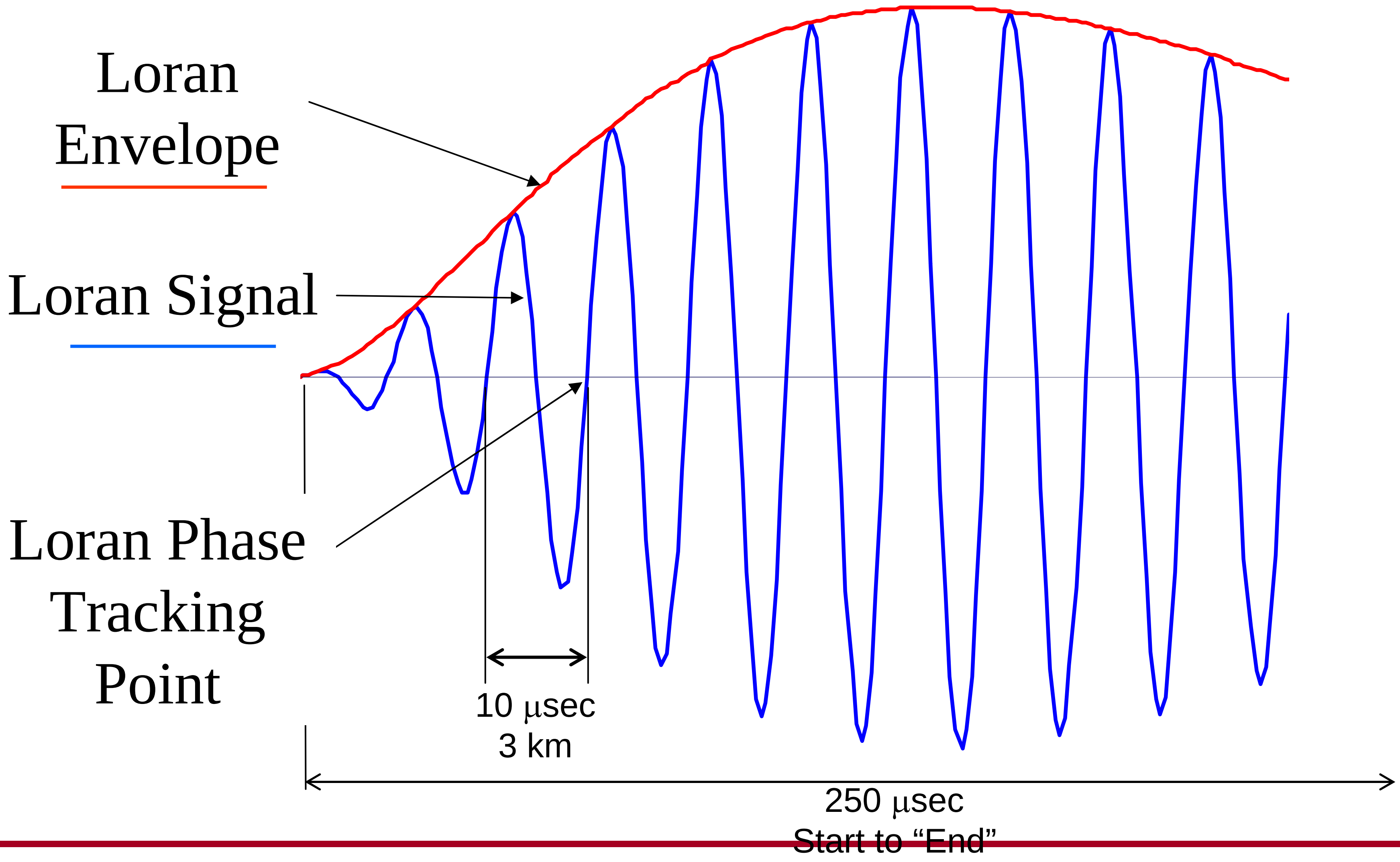
Outline

- Background: WSSE & Cycle Integrity Algorithm
- Two weighting choices
- Integrity of each choice
- Performance
  - Availability
  - Receiver complexity
- Conclusions & Future Work

Loran Cycle & Phase



Purpose

- Key determinant of Loran RNP 0.3 availability is cycle selection
  - Cycle selection algorithm based on WSSE (when we cannot get 3 “trusted” signals)
- Open Issues
  - Demonstrate that the algorithm has integrity
  - Demonstrate how to best implement the algorithm
    - Availability
    - Receiver Implementation

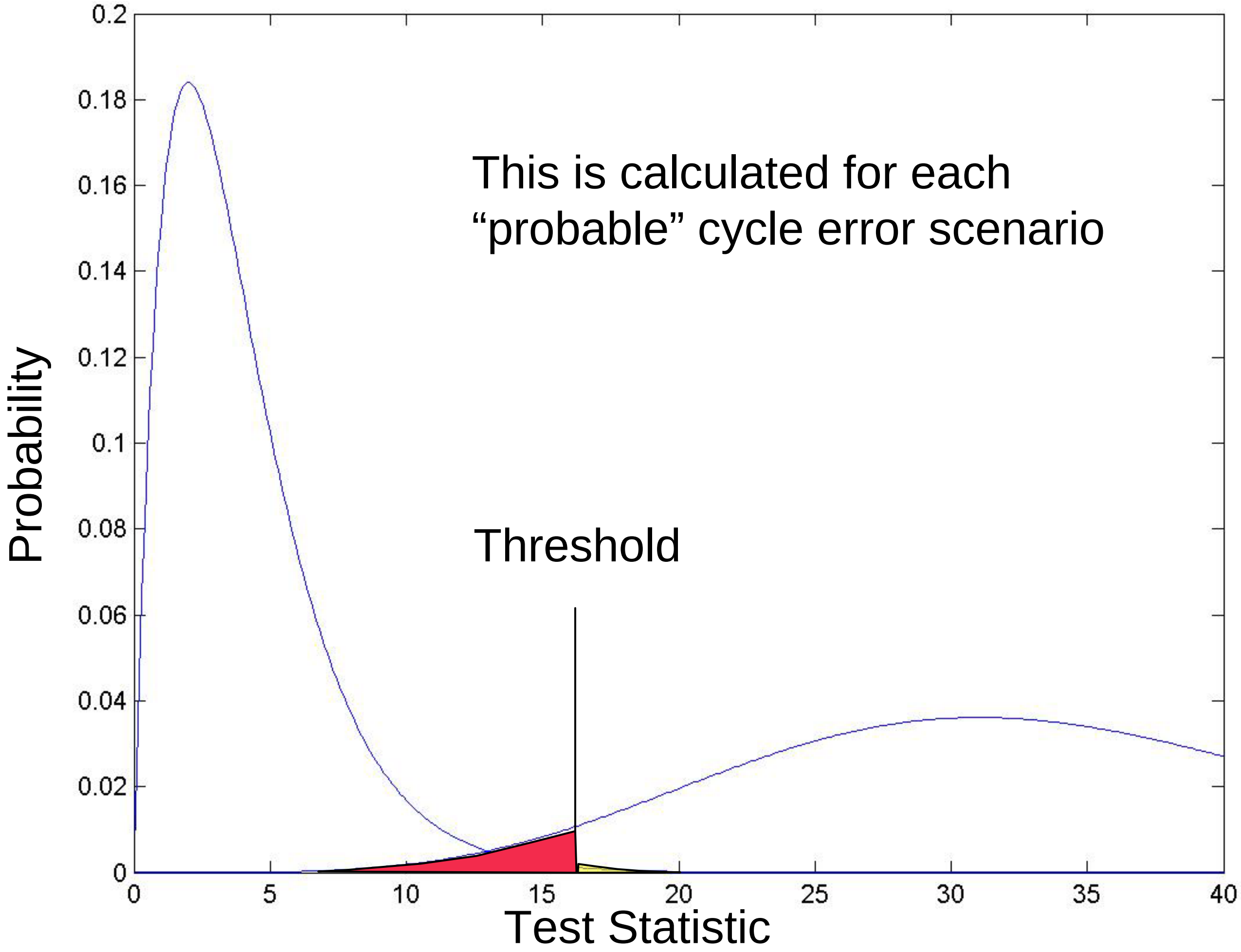
Cycle Identification Integrity Overview

- N=3 Cycle Identification Integrity based on probability of incorrect cycle selection on each signal
  - Probability of incorrect cycle based on ECD bias and noise (SNR)
- N>3 Cycle Identification Integrity algorithm using redundant measurements
  - WSSE statistic provides indication of cycle error

Using Redundant Measurements

- Equations given in other papers & presentations
- WSSE statistic is combination of square of the residuals weighted by a weighting matrix
- The distribution of the WSSE is different if we have a faulted (undetected missed cycle) vs no fault (all cycles correct) condition

Testing the Hypothesis & Determining P_{MD}



- Knowing the no fault (H_0) & faulted (H_1) distribution
  - Allows calc of P_{MD}, P_{FA} for given threshold
- Threshold sets
  - false alarm rate
  - smallest detected error
- Testing No Fault
  - WSSE < threshold -> No Fault
  - Else Faulted
- χ^2 makes test tractable
  - Nominal case is centralized χ^2
  - centralized χ^2 distribution well known

Centralized and Non Centralized χ^2 Distribution

- Centralized χ^2 based on zero mean i.i.d. normal r.v. with σ = 1
- Non-Centralized χ^2 based on non zero mean independent normal r.v. with σ = 1
- The WSSE calculation is meant to get random portion to have uniform distribution (σ = 1)
- Generally, for WSSE to be χ^2
  - $W = R^{-1}$ where R is the error covariance matrix
  - R depends on random (not bias) errors

$Y_i \sim N(0,1)$

$Y_i \sim N(\mu_i, 1)$

$$\chi^2 = \sum_{i=1}^{r} Y_i^2$$

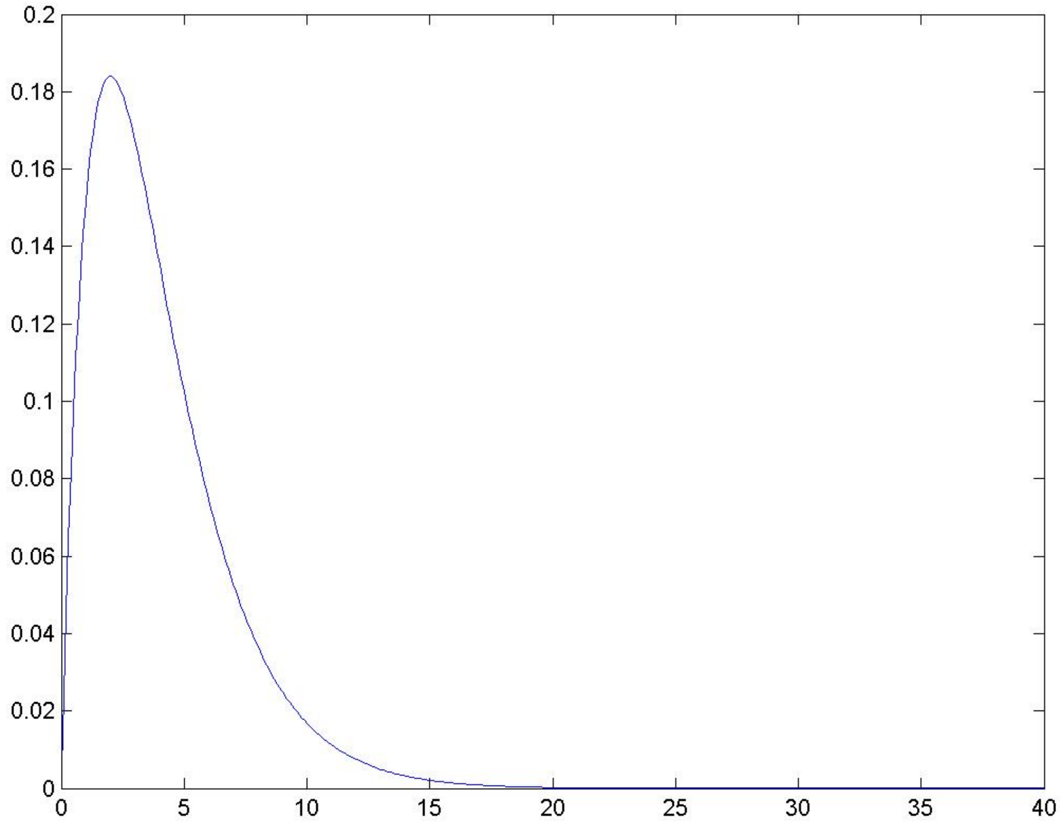
Centralized χ^2 with *r* degrees of freedom

Non centralized χ^2 with *r* degrees of freedom and non centrality parameter λ dependent on μ_i

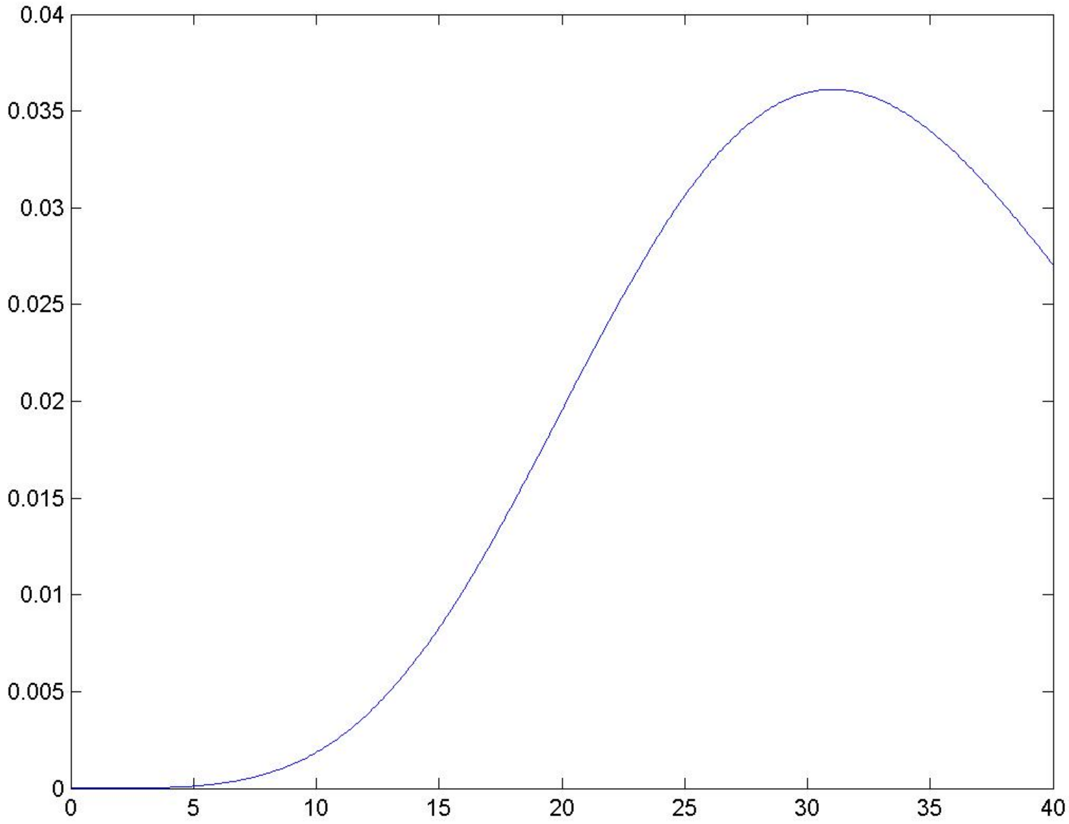
H_o

H_1

Centralized χ^2 Distribution (Unbiased Error)



Non Centralized χ^2 Distribution (Biased Error)



Weighting Matrix

- Weighting matrix affects
  - Ability/confidence of detection
  - Distribution of WSSE
- Weighting matrix for cycle selection does not have to be the same as for position calculation
- Examine two cases
  - Weight by random and bias error (current)
  - Weight by random error only

1. Weight with Random & Bias Error

- Diagonal term is based on noise, correlated & uncorrelated temporal ASF, spatial ASF

$$\boxed{\sigma_{all} = \sigma_{noise} + b_{cor\_tempASF} + b_{uncor\_tempASF} + b_{spatialASF}}$$

- Off diagonal = correlated temporal ASF
- We assumed that, if the cycles are correct, the WSSE is centralized χ^2

$$\boxed{\textit{Residuals } \varepsilon \sim N(0, \sigma_{all})}$$

2. Weight with Random Error

- Diagonal term is based on noise, correlated & uncorrelated temporal ASF, spatial ASF

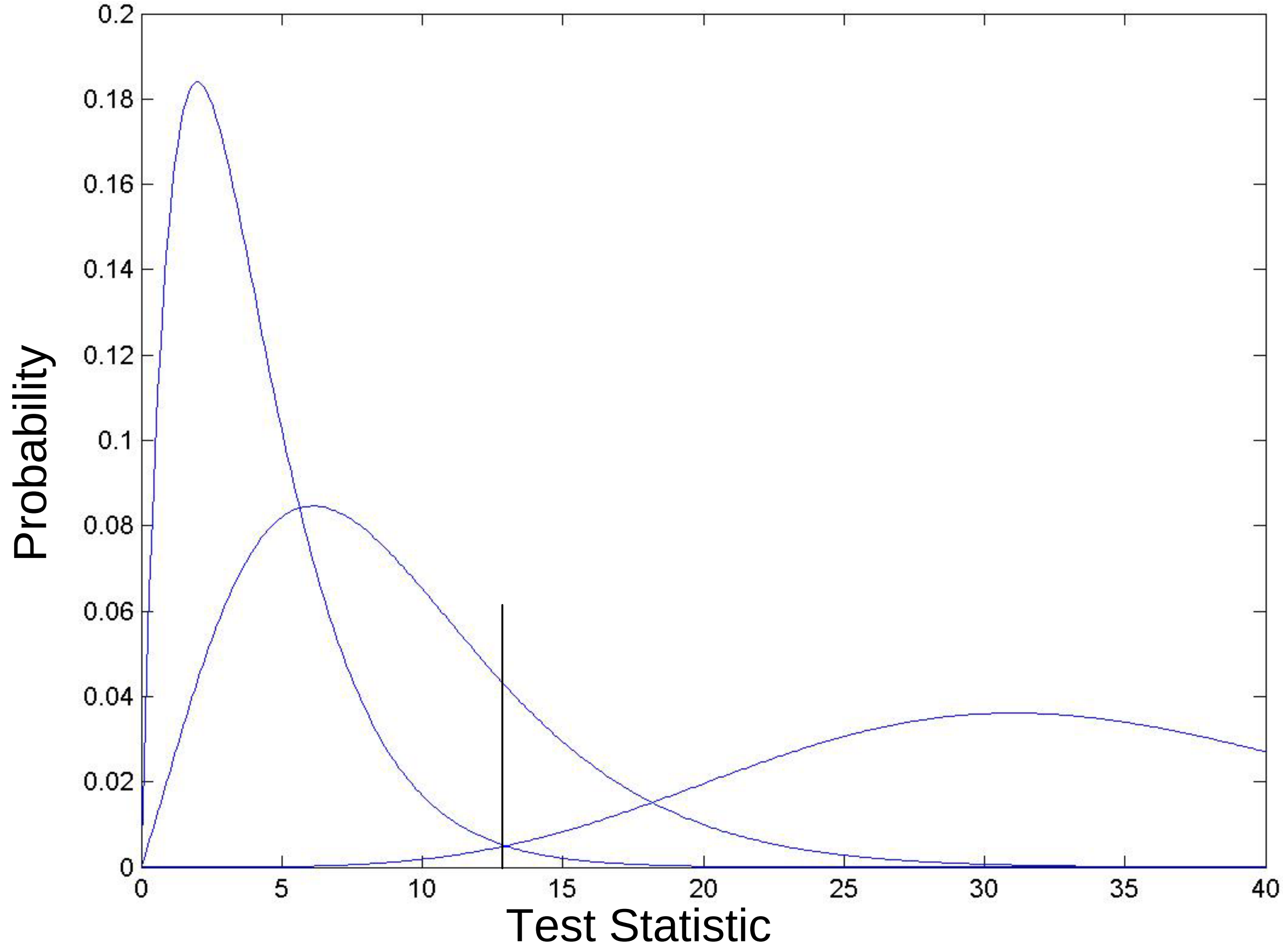
$$\boxed{\sigma_{all} = \sigma_{noise}}$$

- No off diagonal
- We assume that, if the cycle is correct, the WSSE is non centralized χ^2

$$\boxed{\varepsilon \sim N(b_{all}, \sigma_{all})}$$

$$\boxed{b_{all} = b_{cor\_tempASF} + b_{uncor\_tempASF} + b_{spatialASF}}$$

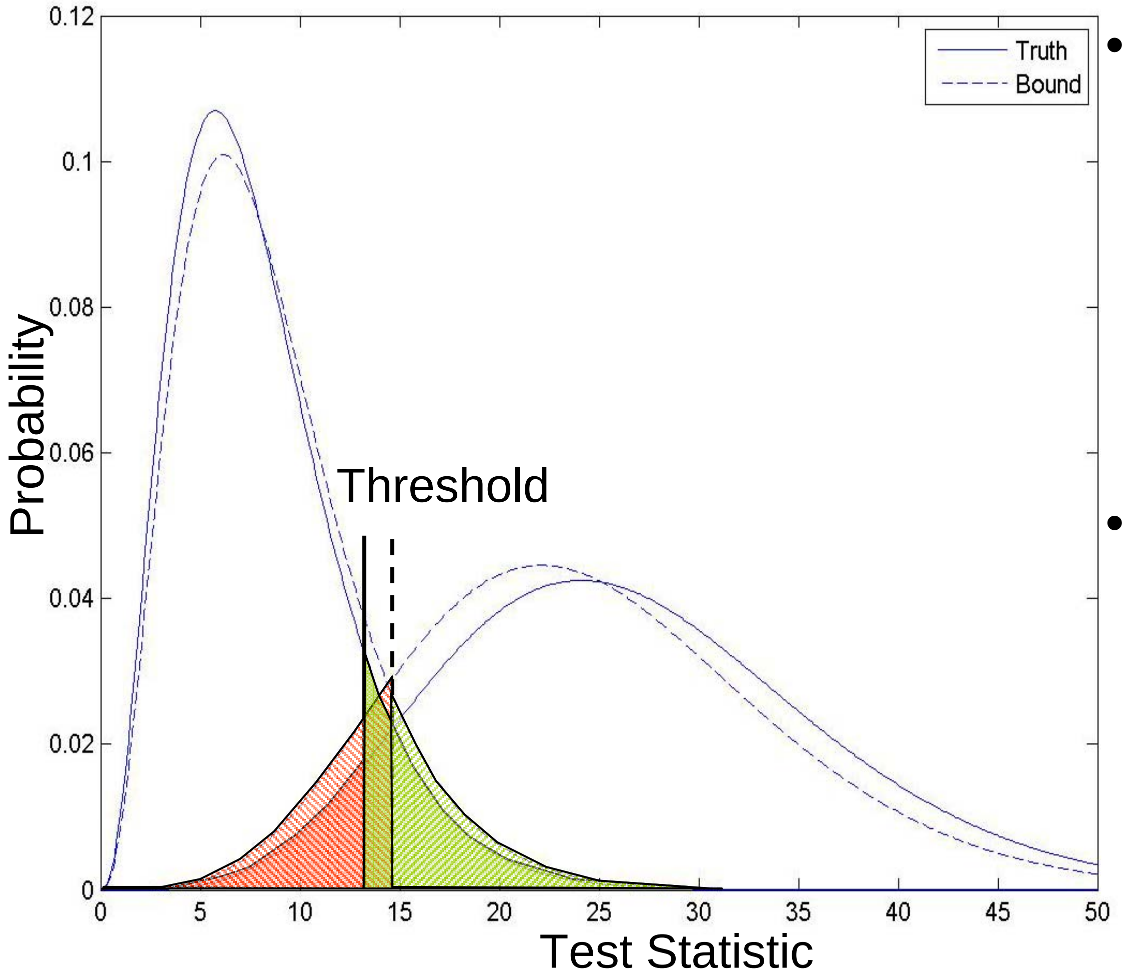
Comparison of Distribution



Use more accurate measure of H_0 based on known biases

Integrity
(How well does the model bound?)

Achieving Integrity



- Bound right tail of no fault distribution – overestimate probability of false alarm
  - Results in a higher threshold than necessary
  - $P_{FA,true} < P_{FA,desired}$
- Bound left tail of faulted distribution – overestimate probability of missed detection
  - Probability of missed detection calculated is larger than true
  - $P_{MD,bound\ calc} > P_{MD,true}$

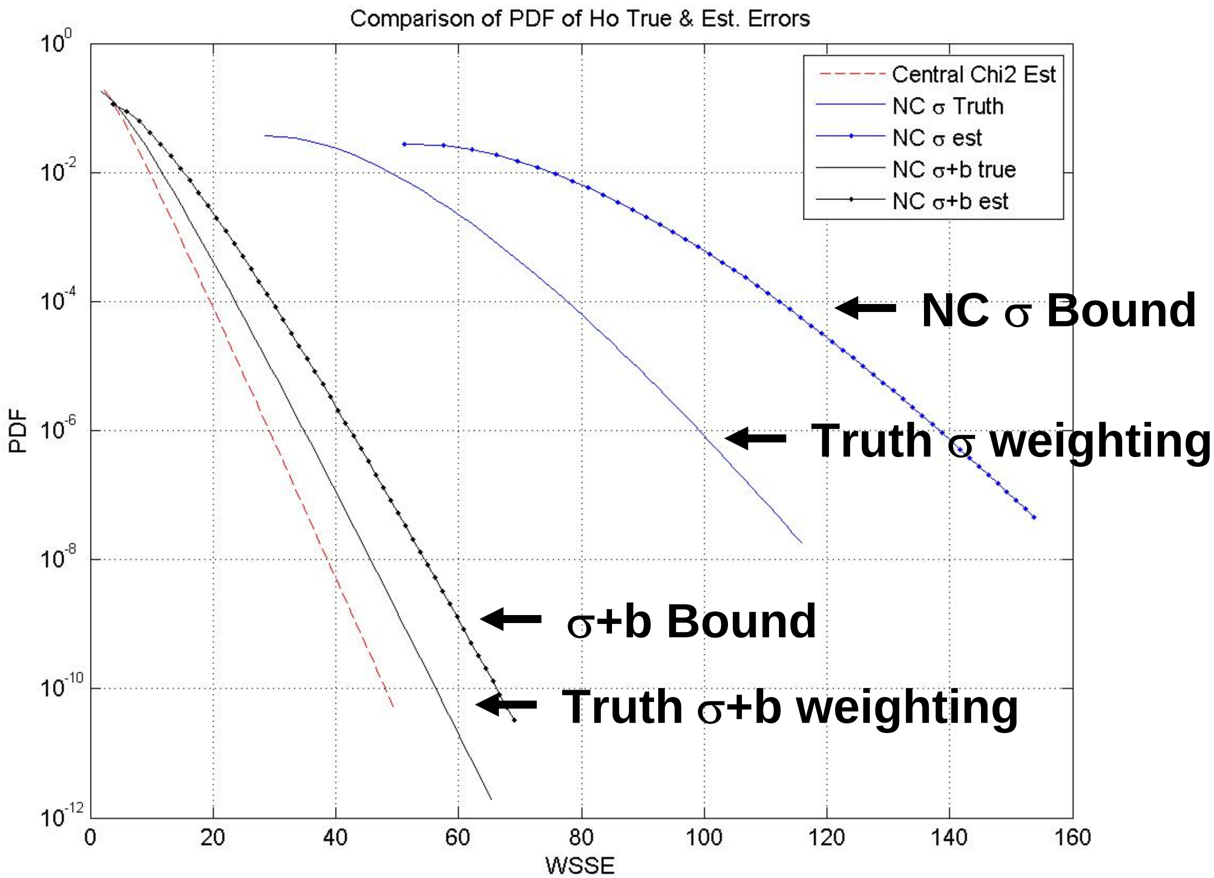
Achieving Bounds on the Distribution

- Use χ^2 distribution to model and bound
  - Mathematically tractable, implementable in receiver
- Caveats:
  - Good estimate of random errors but do not know true biases
    - Biases affect the non centrality parameter
    - Use bias bounds to create bounding χ^2 distributions
  - Adding bias terms to weighting
    - WSSE does not achieve χ^2
    - Can result in smaller WSSE values (in no fault and faulted case

Details for later papers

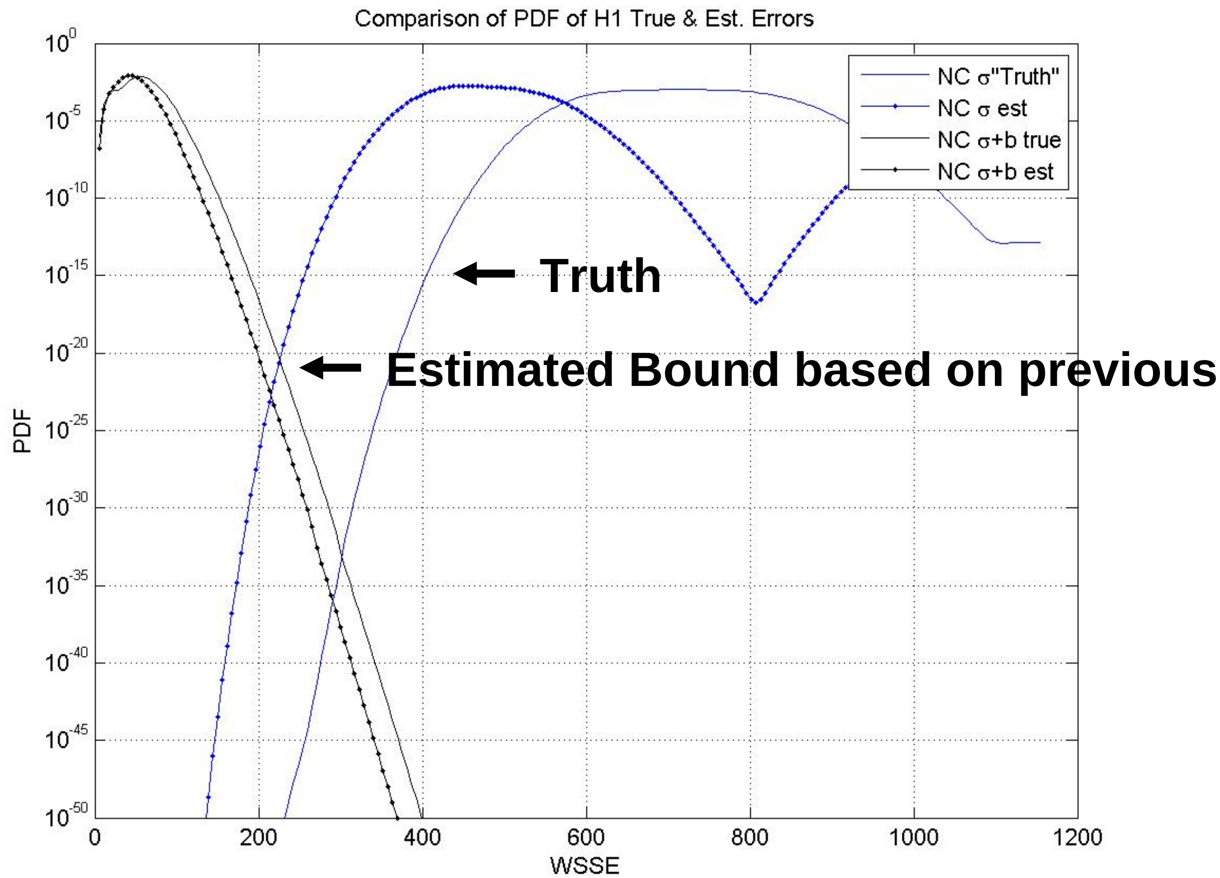
- Can bound the no fault distribution at the desired level (false alarm)
  - Provable for all cases of σ weighting
    - Proper choice of signs for bias bounds
  - Provable for all CONUS geometry of σ+b weighting
- Bound for faulted distribution achievable
  - Provable for all cases of σ weighting
    - Proper choice of signs for bias bounds but overconservative
  - Cannot show that it bounds at the levels desired (10^{-7}) for σ+b weighting

PDF of No Fault Case (different combination of nominal bias)



Estimated distrib. chooses median nc parameter combination of bias

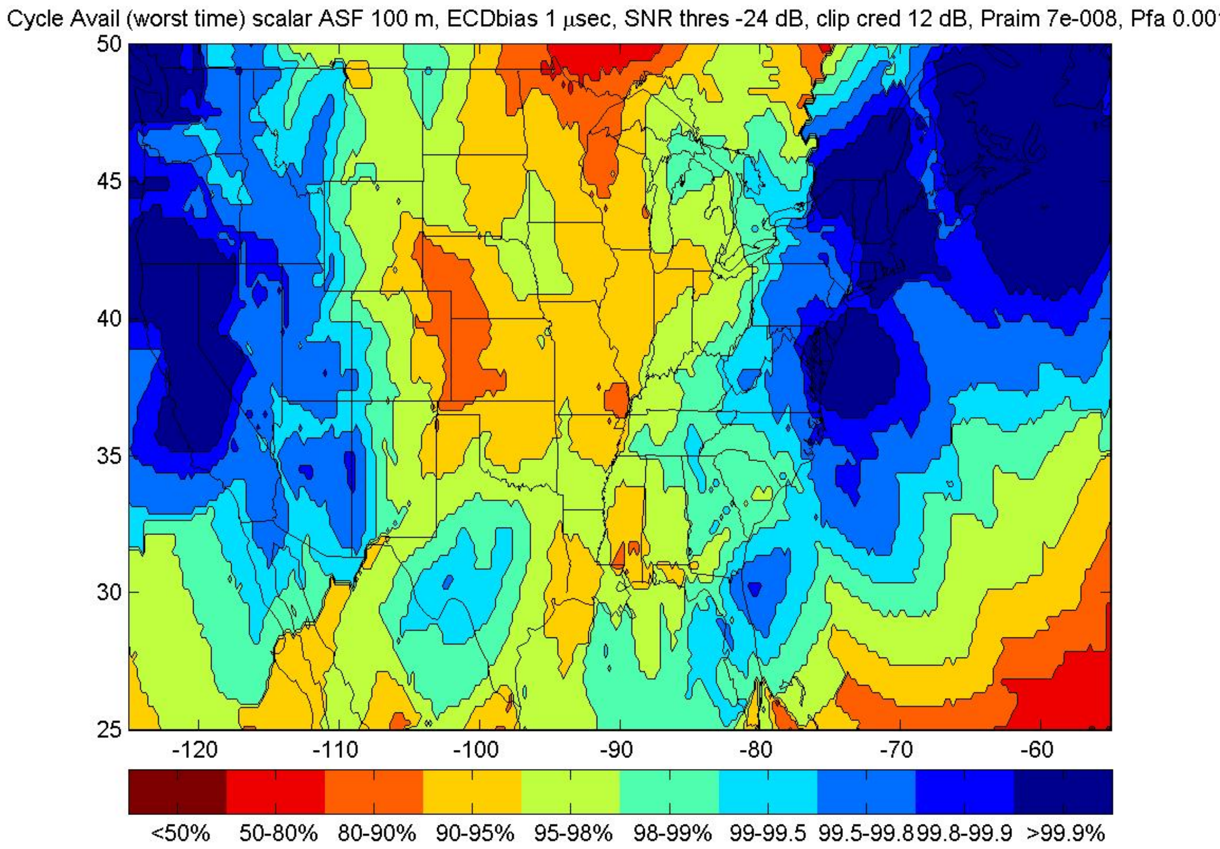
PDF of Faulted Case (different combination of nominal bias)



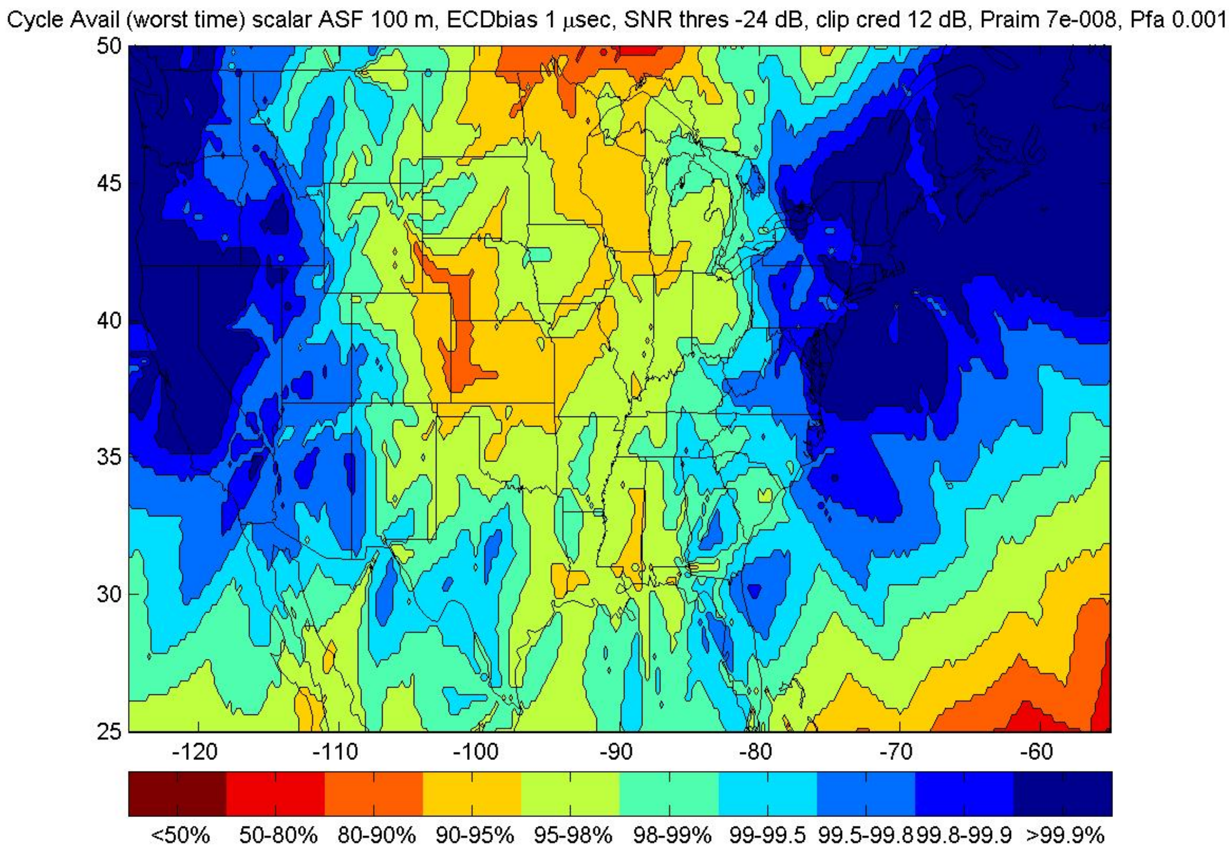
Estimated distributions use nc parameter from no fault case

Availability

Cycle Availability (σ+b)



Cycle Availability (σ)



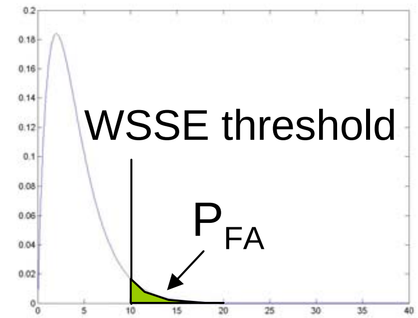
Results are optimistic since it still assumes centralized χ^2 for Ho

Implementation

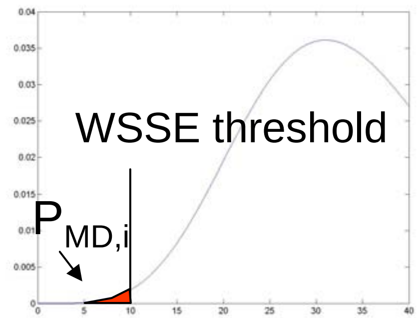
1. Determine parameters for bounding Ho, H1 distribution

Ho: λ_{Ho}, dof
H1: λ_{H1}, dof

2. Calculate WSSE threshold value for desired P_{FA} from Ho distribution



3. Calculate overall P_{MD} summing P_{MD} for every incorrect cycle (H1) case



4. Is $P_{MD} < 7\text{x}10^{-8}$ ($P_{cycle,i}$ derived from SNR)

$P_{MD}=\Sigma P_{cycle,i} P_{MD,i}$

NO → Inadequate Cycle Confidence: Not Available

YES → 5. Is WSSE < WSSE threshold (from 2)?

NO → Inadequate Cycle Confidence: Not Available

YES → Good Cycle Selection

Implications

- Calculating threshold from no fault distribution
  - If central χ^2, the threshold depends solely on number of stations (~ O(10) values)
  - If non central χ^2, the threshold must be calculated after calculating non centrality parameter (O(10^3) values)
- Calculating P_{MD} from non central χ^2
  - Requires O(10^3) values
- Results
  - O(10^6 or 10^7) values stored for σ weighting
  - O(10^3 or 10^4) values stored for σ+b weighting

Conclusions & Thoughts

- For WSSE χ^2 assumptions to be met, weighting should be based on σ only
- Both σ and σ+b weighting may be usable
  - Former has better availability, more provable integrity
  - Latter is easier to implement
- More work needs to be done to reduce conservatism on σ
  - Implement in new version in coverage code

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- The views expressed herein are those of the presenter and are not to be construed as official or reflecting the views of the U.S. Coast Guard, Federal Aviation Administration, or Department of Transportation .